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Matter coupled $F(4)$ supergravity and the AdS_6/CFT_5 correspondence

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Abstract: $F(4)$ supergravity, the gauge theory of the exceptional six-dimensional Anti-de Sitter superalgebra, is coupled to an arbitrary number of vector multiplets whose scalar components parametrize the quaternionic manifold $\mathrm{SO}(4,n)/\mathrm{SO}(4) \times \mathrm{SO}(n)$. By gauging the compact subgroup $\mathrm{SU}(2)_d \otimes \mathcal{G}$, where $\mathrm{SU}(2)_d$ is the diagonal subgroup of $\mathrm{SO}(4) \simeq \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ (the$R$-symmetry group of six-dimensional Poincaré supergravity) and $\mathcal{G}$ is a compact group such that $\dim \mathcal{G} = n$, we compute the scalar potential which, besides the gauge coupling constants, also depends in non-trivial way on the parameter m associated to a massive 2-form $B_{\mu\nu}$ of the gravitational multiplet. The potential admits an AdS background for $g = 3m$, as the pure $F(4)$-supergravity. We compute the scalar squared masses (which are all negative) and retrieve the results dictated by AdS_6/CFT_5 correspondence from the conformal dimensions of boundary operators. The boundary $F(4)$ superconformal fields are realized in terms of a singleton superfield (hypermultiplet) in harmonic superspace with flag manifold $\mathrm{SU}(2)/\mathrm{U}(1) = S^2$. We analize the spectrum of short representations in terms of superconformal primaries and predict general features of the K-K specrum of massive type-IIA supergravity compactified on warped $AdS_6 \otimes S^4$.

Keywords: Supergravity Models, AdS/CFT Correspondance, Differential and Algebraic Geometry.

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Contents

1. Introduction

In the past two years much work has been devoted to the verification of the conjectured duality between AdS_{d+1} supergravities and conformal field theories in Minkowski d-dimensional space [1] (for an extensive review of the topic see [2]). In this framework an important check is given by comparing the Kaluza-Klein mass spectrum of compactified M and/or superstring theories in lower dimensions, with the mass spectrum computed in terms of the scale dimension $\Delta \equiv E_0$ of the conformal operators of the dual theory. While much effort has been devoted to the investigation of such correspondence for seven, five- and four-dimensional supergravities, the six-dimensional case based on the $F(4)$ exceptional supergroup [3, 4] has been studied only in ref. [5]. In this paper it was shown that the $F(4)$ theory has on its boundary a superconformal field theory at a fixed point of the five-dimensional Yang-Mills theory. The relevant superconformal operators in five dimensions, which are dual to the supergravity states in $D = 6$, are built in terms of the singleton representation of $F(4) \otimes \mathcal{G}$ that is just a 5-dimensional hypermultiplet carring a representation of the flavour group $\mathcal{G}$.

In this way, it is possible to predict the mass spectrum of the “massless” supergravity states in a supersymmetric AdS background from the corresponding E_0 quantum number of the conformal supermultiplet. This prediction, however, could not be checked since the matter coupled $F(4)$ theory was not yet constructed; what one usually refers to as $F(4)$ supergravity is the theory constructed by Romans [6] in the pure supergravity case, that is, without matter coupling. (Some aspects of the $F(4)$ theory have also been discussed in the framework of dimensional reduction from seven dimensions in [7].)

Other interesting results on the $F(4)$ theory were obtained in ref. [8] where it was shown that the theory obtained from spontaneous compactification of Massive type-IIA supergravity in ten dimensions down to the $F(4)$ gauged supergravity in six dimensions can be described in terms of the near horizon geometry of the D4-D8 brane system. Furthermore in ref. [9] it was shown that the $F(4)$ gauged pure supergravity constructed in ref. [6], can be obtained as a consistent warped reduction on S^4 from massive type-IIA ten-dimensional supergravity [10].

The AdS_6/CFT_5 correspondence further predicts that the $K-K$ excitations of massive type IIA on $AdS_6 \otimes S^4$ should be related to towers of superconformal primary operators of the boundary superconformal field theory. The latter having only eight Poincaré supercharges is not completely fixed by supersymmetry and in fact it can have various global symmetry groups $\mathcal{G}$ which were classified in [11]. This reflects in the gauge groups of the vector multiplets of $F(4)$ whose coupling is discussed here.

It will be shown that the massless graviton (stress-energy tensor) multiplet and the "current multiplet" related to the $\mathcal{G}$ gauge fields, previously discussed in [5], are actually the first members of two distinct towers of (short) conformal superfields, one corresponding to the 1/2 BPS multiplets (massive vector multiplets), the other corresponding to the graviton recurrences.

Interestingly enough they correspond to two isolated classes of highest-weight UIR's of the $F(4)$ superalgebra, separated from the continuous spectrum (in E_0) suggested to exist in [12].

The paper is organized as follows:

In section 2 the geometric approach, and the closely related superspace solutions of Bianchi identities are presented in the framework of the pure $F(4)$ supergravity of ref. [6]. This, besides to give the right geometrical setting for the matter coupling and gauging of the theory, allows us to discuss in a simple way the algebraic foundation of the peculiar properties of $F(4)$ superalgebra, namely the relations between the SU(2) gauging coupling constant g and the inverse AdS radius $4m$ and the related "Higgs phenomenon" by which the gravitational two-form $B_{\mu\nu}$ becomes massive.

In section 3 the geometrical framework developed in section 2 is applied to the matter coupling and in section 4 the gauging of $D=6$, $N=(1,1)$ supergravity is studied.

In section 5 the scalar potential is derived and it is shown to admit an AdS_6 supersymmetric vacuum.

In section 6 the boundary conformal field theory and its short representations in terms of harmonic superfields [13] (with harmonic space $S^2 = \mathrm{SU}(2)/\mathrm{U}(1)$) are presented and the spectrum of K-K excitations predicted.

2. The geometrical approach

In this section we set up a suitable framework for the discussion of the matter coupled $F(4)$ supergravity theory and its gauging. This will allow us to set up the formalism for the matter coupling in the next section. Actually we will just give the essential definitions of the Bianchi identities approach in superspace , while all the relevant results, specifically the supersymmetry transformation laws of the fields, will be given in the ordinary space-time formalism.

First of all it is useful to discuss the main results of ref. [6] by a careful study in superspace of the Poincaré and AdS supersymmetric vacua. Let us recall the content of $D = 6$, $N = (1, 1)$ supergravity multiplet:

$$\left(V^a_\mu, A^\alpha_\mu, B_{\mu\nu}, \psi^A_\mu, \psi^{\dot{A}}_\mu, \chi^A, \chi^{\dot{A}}, e^\sigma\right), \tag{2.1}$$

where V^a_μ is the six-dimensional vielbein, $\psi^A_\mu, \psi^{\dot{A}}_\mu$ are left-handed and right-handed four-component gravitino fields, respectively, A and $\dot{A}$ transforming under the two factors of the R-symmetry group $\mathrm{O}(4) \simeq \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$, $B_{\mu\nu}$ is a 2-form, A^α_μ ($\alpha = 0, 1, 2, 3$), are vector fields, $\chi^A, \chi^{\dot{A}}$ are left-handed and right-handed spin $\frac{1}{2}$ four components dilatinos, and e^σ denotes the dilaton.

Our notations are as follows: $a, b, \ldots = 0, 1, 2, 3, 4, 5$ are Lorentz flat indices in $D = 6$ $\mu, \nu, \ldots = 0, 1, 2, 3, 4, 5$ are the corresponding world indices, $A, \dot{A} = 1, 2$. Moreover our metric is $(+, -, -, -, -, -)$.

We recall that the description of the spinors of the multiplet in terms of left-handed and right-handed projection holds only in a Poincaré background, while in an AdS background the chiral projection cannot be defined and we are bounded to use 8-dimensional pseudo-Majorana spinors. In this case the R-symmetry group reduces to the $\mathrm{SU}(2)$ subgroup of $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$, the R-symmetry group of the chiral spinors. For our purposes, it is convenient to use from the very beginning 8-dimensional pseudo-Majorana spinors even in a Poincaré framework, since we are going to discuss in a unique setting both Poincaré and AdS vacua.

The pseudo-Majorana condition on the gravitino 1-forms is as follows:

$$(\psi_A)^\dagger \gamma^0 = \overline{(\psi_A)} = \epsilon^{AB} {\psi_B}^t, \tag{2.2}$$

where we have chosen the charge conjugation matrix in six dimensions as the identity matrix (an analogous definition holds for the dilatino fields). We use eight-dimensional antisymmetric gamma matrices, with $\gamma^7 = i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^4\gamma^5$, which implies $\gamma_7^T = -\gamma_7$ and $(\gamma_7)^2 = -1$. The indices $A, B, \ldots = 1, 2$, of the spinor fields ψ_A, χ_A transform in the fundamental of the diagonal subgroup $\mathrm{SU}(2)$ of $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$. For a generic $\mathrm{SU}(2)$ tensor T, raising and lowering of indices are defined by

$$\begin{aligned} T^{\ldots A \ldots} &= \epsilon^{AB} \; T^{\ldots}{}_{B}{}^{\ldots}, \\ T_{\ldots A \ldots} &= T_{\ldots}{}^{B}{}_{\ldots} \; \epsilon_{BA}. \end{aligned} \tag{2.3}$$

To study the supersymmetric vacua let us write down the Maurer-Cartan Equations (M.C.E.) dual to the $F(4)$ Superalgebra (anti)commutators:

$$\mathcal{D}V^a - \frac{i}{2}\ \overline{\psi}_A\gamma_a\psi^A = 0\,, \tag{2.4}$$

$$\mathcal{R}^{ab} + 4m^2\ \ V^aV^b + m\overline{\psi}_A\gamma_{ab}\psi^A = 0\,, \tag{2.5}$$

$$dA^r + \frac{1}{2}\ g\ \epsilon^{rst}A_sA_t - i\ \overline{\psi}_A\psi_B\ \sigma^{rAB} = 0\,, \tag{2.6}$$

$$D\psi_A - im\gamma_a\psi_AV^a = 0\,. \tag{2.7}$$

Here $V^a, \omega^{ab}, \psi_A, A^r, (r=1,2,3)$, are superfield 1-forms dual to the $F(4)$ supergenerators which at $\theta = 0$ have as dx^μ components

$$V^a_\mu = \delta^a_\mu\,, \qquad \psi_{A\mu} = A^r_\mu = 0\,, \qquad \omega^{ab}_\mu = \text{pure gauge}\,. \tag{2.8}$$

Furthermore $\mathcal{R}^{ab} \equiv d\omega^{ab} - \omega^{ac}\wedge\ \omega_c{}^b$, $\mathcal{D}$ is the Lorentz covariant derivative, D is the $\mathrm{SO}(1,5)\otimes\mathrm{SU}(2)$ covariant derivative, which on spinors acts as follows:

$$D\psi_A \equiv d\psi_A - \frac{1}{4}\gamma_{ab}\omega^{ab}\psi_A - \frac{i}{2}\sigma^r_{AB}A_r\psi^B\,. \tag{2.9}$$

Note that $\sigma^{rAB} = \epsilon^{BC}\sigma^{rA}{}_C$, where $\sigma^{rA}{}_B\ \ (r=1,2,3)$ denote the usual Pauli matrices, are symmetric in $A,\ B$.

Let us point out that the $F(4)$ superalgebra, despite the presence of two different physical parameters, the $\mathrm{SU}(2)$ gauge coupling constant g and the inverse AdS radius m, really depends on just one parameter since the closure under d-differentiation of eq. (2.7) (equivalent to the implementation of Jacobi identities on the generators), implies $g = 3m$; to recover this result one has to use the following Fierz identity involving 3-ψ_A's 1-forms:

$$\frac{1}{4}\gamma_{ab}\psi_A\overline{\psi}_B\gamma^{ab}\psi_C\epsilon^{AC} - \frac{1}{2}\gamma_a\psi_A\overline{\psi}_B\gamma^a\psi_C\epsilon^{AC} + 3\psi_C\overline{\psi}_B\psi_A\epsilon^{AC} = 0\,. \tag{2.10}$$

The $F(4)$ superalgebra described by eqs. (2.4)–(2.7) fails to describe the physical vacuum because of the absence of the superfields 2-form B and 1-form A^0 whose space-time restriction coincides with the physical fields $B_{\mu\nu}$ and A^0_μ appearing in the supergravity multiplet. The recipe to have all the fields in a single algebra is well known and consists in considering the Free Differential Algebra (F.D.A.) [14] obtained from the $F(4)$ M.C.E.'s by adding two more equations for the 2-form B and for the 1-form A^0 (the 0-form fields χ_A and σ do not appear in the algebra since they are set equal to zero in the vacuum). It turns out that to have a consistent F.D.A. involving B and A^0 one has to add to the $F(4)$ M.C.E.'s two more equations involving dA^0 and dB; in this way one obtains an extension of the M.C.E's to the following F.D.A.:

$$\mathcal{D}V^a - \frac{i}{2}\,\overline{\psi}_A\gamma_a\psi^A = 0\,, \tag{2.11}$$

$$\mathcal{R}^{ab} + 4m^2\; V^aV^b + m\overline{\psi}_A\gamma_{ab}\psi^A = 0\,, \tag{2.12}$$

$$dA^r + \frac{1}{2}\,g\;\epsilon^{rst}A_sA_t - i\;\overline{\psi}_A\psi_B\;\sigma^{rAB} = 0\,, \tag{2.13}$$

$$dA^0 - mB - i\;\overline{\psi}_A\gamma_7\psi^A = 0\,, \tag{2.14}$$

$$dB + 2\;\overline{\psi}_A\gamma_7\gamma_a\psi^AV^a = 0\,, \tag{2.15}$$

$$D\psi_A - im\gamma_a\psi_AV^a = 0\,. \tag{2.16}$$

Equations (2.14) and (2.15) were obtained by imposing that they satisfy the d-closure together with eqs. (2.11). Actually the closure of (2.15) relies on the 4-ψ_A's Fierz identity

$$\overline{\psi}_A\gamma_7\gamma_a\psi_B\epsilon^{AB}\overline{\psi}_C\gamma^a\psi_D\epsilon^{CD} = 0\,. \tag{2.17}$$

The interesting feature of the F.D.A. (2.11)–(2.16) is the appearance of the combination $dA^0 - mB$ in (2.14). That means that the dynamical theory obtained by gauging the F.D.A. out of the vacuum will contain the fields A^0_μ and $B_{\mu\nu}$ always in the single combination $\partial_{[\mu}A^0_{\nu]} - mB_{\mu\nu}$. At the dynamical level this implies, as noted by Romans [6], an Higgs phenomenon where the 2-form B "eats" the 1-form A^0 and acquires a non-vanishing mass m.[1]

In summary, we have shown that two of the main results of [6], namely the existence of an AdS supersymmetric background only for $g = 3m$ and the Higgs-type mechanism by which the field $B_{\mu\nu}$ becomes massive acquiring longitudinal degrees of freedom in terms of the the vector A^0_μ, are a simple consequence of the algebraic structure of the F.D.A. associated to the $F(4)$ supergroup written in terms of the vacuum-superfields.

It is interesting to see what happens if one or both the parameters g and m are zero. Setting $m = g = 0$, one reduces the $F(4)$ superalgebra to the $D = 6\;\; N = (1,1)$ superalgebra existing only in a super Poincaré background; in this case the four-vector $A^\alpha \equiv (A^0, A^r)$ transforms in the fundamental of the R-symmetry group SO(4) while the pseudo-Majorana spinors ψ_A, χ_A can be decomposed in two chiral spinors in such a way that all the resulting F.D.A. is invariant under SO(4).

Furthermore it is easy to see that no F.D.A. exists if either $m = 0,\ g \neq 0$ or $m \neq 0,\ g = 0$, since the corresponding equations in the F.D.A. do not close anymore under d-differentiation. In other words the gauging of SU(2), $g \neq 0$ must be necessarily accompanied by the presence of the parameter m which, as we have seen, makes the closure of the supersymmetric algebra consistent for $g = 3m$.

Let us now consider the dynamical theory out of the vacuum. For the sake of completeness and in order to establish a consistent notation, we first reconsider

[1] An analogous phenomenon takes place also in $D = 5$; see[15].

the pure supergravity theory of ref. [6] in a superspace formalism. We start from the F.D.A. of $D = 6$, $N = (1,1)$, SU(2)-gauged Poincaré supergravity, namely we consider the F.D.A. (2.11)–(2.16) taking $m = 0$, $g \neq 0$.

Following the geometrical approach [14], we define the supercurvatures as deviation from the $m = 0$, $g \neq 0$ F.D.A. as follows:

$$T^a = \mathcal{D}V^a - \frac{i}{2}\ \overline{\psi}_A \gamma_a \psi^A V^a = 0\,, \tag{2.18}$$
$$R^{ab} = \mathcal{R}^{ab}\,, \tag{2.19}$$
$$H = dB + 2e^{-2\sigma}\ \overline{\psi}_A \gamma_7 \gamma_a \psi^A V^a\,, \tag{2.20}$$
$$F = dA - ie^{\sigma}\ \overline{\psi}_A \gamma_7 \psi^A\,, \tag{2.21}$$
$$F^r = dA^r + \frac{1}{2}\ g\ \epsilon^{rst} A_s A_t - ie^{\sigma}\ \overline{\psi}_A \psi_B\ \sigma^{rAB}\,, \tag{2.22}$$
$$\rho_A = D\psi_A\,, \tag{2.23}$$
$$R(\chi_A) \equiv D\chi_A\,, \tag{2.24}$$
$$R(\sigma) \equiv d\sigma\,, \tag{2.25}$$

where we have implemented the kinematical constraint that the supertorsion T^a is identically zero, and we have added the "curvatures" of the 0-forms χ_A and σ defined as their (covariant) differential.

Differentiating the curvatures one obtains the Bianchi identities

$$R^{ab}V_b - i\overline{\psi}_A \gamma^a \rho^A = 0\,, \tag{2.26}$$
$$\mathcal{D}R^{ab} = 0\,, \tag{2.27}$$
$$dH + 4e^{-2\sigma} d\sigma\ \overline{\psi}_A \gamma_7 \gamma_a \psi^A V^a + 4e^{-2\sigma} \overline{\psi}_A \gamma_7 \gamma_a \rho^A V^a = 0\,, \tag{2.28}$$
$$DF + id\sigma e^{\sigma}\ \overline{\psi}_A \gamma_7 \psi^A - 2ie^{\sigma}\ \overline{\psi}_A \gamma_7 \rho^A = 0\,, \tag{2.29}$$
$$DF^r + id\sigma e^{\sigma}\ \overline{\psi}_A \psi_B\ \sigma^{rAB} - 2ie^{\sigma}\ \overline{\psi}_A \rho_B\ \sigma^{rAB} = 0\,, \tag{2.30}$$
$$D^2\psi_A + \frac{1}{4} R^{ab} \gamma_{ab} \psi_A - \frac{i}{2}\ g\ \sigma_{rAB} F^r \psi^B = 0\,, \tag{2.31}$$
$$D^2\chi_A + \frac{1}{4} R^{ab} \gamma_{ab} \chi_A - \frac{i}{2}\ g\ \sigma_{rAB} F^r \chi^B = 0\,, \tag{2.32}$$
$$d^2\sigma = 0\,. \tag{2.33}$$

The superspace solutions of the previous Bianchi Identities are given by:

$$R^{ab} = \widetilde{R}^{ab}_{cd} V^c V^d + \overline{\theta}^{ab}_{cA} \psi^A V^c + \overline{\psi}_A M^{ab} \psi^A + \frac{1}{4} g\ e^{\sigma} \overline{\psi}_A \gamma^{ab} \psi^A\,, \tag{2.34}$$
$$H = \widetilde{H}_{abc} V^a V^b V^c + 4i\ e^{-2\sigma}\ \ \overline{\psi}_A \gamma_7 \gamma_{ab} \chi^A\ \ V^a V^b\,, \tag{2.35}$$
$$F = \widetilde{F}_{ab} V^a V^b + 2\ e^{\sigma}\ \ \overline{\psi}_A \gamma_7 \gamma_a \chi^A V^a\,, \tag{2.36}$$
$$F^r = \widetilde{F}^r_{ab}\ V^a V^b + 2\ e^{\sigma}\ \overline{\psi}_A \gamma_a \chi_B\ \ \sigma^{rAB}\ V^a\,, \tag{2.37}$$
$$D\psi_A = \widetilde{D_{[a}\psi_{b]A}} V^a V^b + \frac{1}{16} e^{-\sigma} [\epsilon_{AB}\ \widetilde{F}_{ab} \gamma_7 - \sigma_{rAB}\ \widetilde{F}^r_{ab}] (\gamma^{cab} - 6\delta^{ca}\gamma^b)\ \psi^B V_c +$$

$$+ \frac{i}{32} e^{2\sigma} \widetilde{H}_{abc}\, \gamma_7 (\gamma^{dabc} - 3\delta^{ad}\gamma^{bc})\; \psi_A V_d - \frac{i}{4} g\; e^{\sigma} \gamma_a \psi_A V^a +$$
$$+ \frac{1}{4}\gamma_7 \psi_A \overline{\chi}^C \gamma^7 \psi_C - \frac{1}{2}\gamma_a \psi_A \overline{\chi}^C \gamma^a \psi_C + \frac{1}{2}\gamma_7\gamma_a \psi_A \overline{\chi}^C \gamma^7 \gamma^a \psi_C -$$
$$- \frac{1}{8}\gamma_{ab} \psi_A \overline{\chi}^C \gamma^{ab} \psi_C - \frac{1}{8}\gamma_7\gamma_{ab} \psi_A \overline{\chi}^C \gamma^7 \gamma^{ab} \psi_C + \frac{1}{4}\psi_A \overline{\chi}^C \psi_C\,, \tag{2.38}$$
$$D\chi_A = \widetilde{D_a \chi_A} V^a + \frac{i}{2}\gamma^a \widetilde{\partial_a \sigma}\; \psi_A + \frac{i}{16} e^{\sigma} [\epsilon_{AB}\; \widetilde{F}_{ab}\gamma_7 + \sigma_{rAB}\; \widetilde{F}^r_{ab}] \gamma^{ab} \psi^B +$$
$$+ \frac{1}{32} e^{2\sigma} \widetilde{H}_{abc} \gamma_7 \gamma^{abc} \psi_A - \frac{1}{4} g e^{\sigma} \psi_A\,, \tag{2.39}$$
$$d\sigma = \widetilde{\partial_a \sigma} V^a + \overline{\chi}_A \psi^A\,, \tag{2.40}$$

where we have no written down the explicit form of $\overline{\theta}^{ab}_{cA}$ and M^{ab} since they are rather cumbersome and uninteresting for our later purposes.[2] Note that setting $g = 0$ in the fermion fields solutions the corresponding superspace curvatures give the solutions of the Bianchi identities of $N = (1,1)$ $D = 6$ Poincaré supergravity (in this case of course the covariant derivatives in the l.h.s. are Lorentz covariant and the $\mathcal{F}^r$ is abelian).

At $g \neq 0$, however, one can immediately see that the vacuum configuration corresponding to this solution is not supersymmetric. Indeed if we set

$$\widetilde{F}_{ab} = \widetilde{F}^r_{ab} = \widetilde{H}_{abc} = \sigma = \chi_A = \psi_{A\mu} = 0\,, \tag{2.41}$$

we see that the superspace field strengths $D\psi_A$, $D\chi_A$ are not zero in the vacuum if $g \neq 0$, due to the presence of the gauging terms proportional to g in their superspace parametrisations. In the ordinary space-time language that means that, in the vacuum, the supersymmetry transformation law of the dilatino is not zero and that the gravitino does not transforms as the Lorentz covariant derivative of the supersymmetry parameter ε_A:

$$\delta\chi_A = -\frac{1}{4} g e^{\sigma} \varepsilon_A\,, \tag{2.42}$$
$$\delta\psi_{A\mu} = \mathcal{D}_\mu \varepsilon_A - \frac{i}{4} g\; e^{\sigma} \gamma_\mu \varepsilon_A\,. \tag{2.43}$$

This is of course in line with what we have found previously in the study of the supersymmetric vacua, that is the fact that at $g = 0$ we have suitable supersymmetric Poincaré vacuum, while we are bound to modify the previous solution if we want to obtain a supersymmetric AdS vacuum.

This modification however is very simple to obtain since we already know that the proper vacuum configuration of the $F(4)$ theory requires a F.D.A. modified by the mass parameter m suitable related to g, as we have shown in the previous discussion.

[2]Note that the tilded quantities correspond, on space-time, to the supercovariant field strengths: namely e.g. projecting equation (2.35) along the space time differentials dx^μ's we have: $\widetilde{H}_{\mu\nu\rho} = H_{\mu\nu\rho} - 4i\; e^{-2\sigma}\;\; \overline{\psi}_{A[\mu}\gamma_{\nu\rho]}\gamma_7\chi^A$ that is, using definition (2.20) $\widetilde{H}_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]} + 2e^{-2\sigma}\;\overline{\psi}_{A[\mu}\gamma^7\gamma_\rho\psi^A_{\nu]} - 4i\; e^{-2\sigma}\;\; \overline{\psi}_{A[\mu}\gamma_{\nu\rho]}\gamma_7\chi^A$. Analogous formulae hold for the other tilded quantities.

Therefore to obtain such vacuum we modify the r.h.s. of eqs. (2.34)–(2.40) by adding suitable m terms in such a way that the Bianchi identities (2.26)–(2.33) are still satisfied. To study the supersymmetric vacua of the new solutions, it is convenient to denote with the suffix $(g=0)$ the fermionic field strengths and the Lorentz curvature in eqs. (2.34)–(2.40) evaluated at $g=0$. The new solution extending the previous one with terms containing m is given by:

$$\begin{aligned}
R^{ab(\mathrm{new})} &= R^{ab(g=0)}+4m^2\, e^{-6\sigma}V^aV^b+\frac{1}{4}me^{-3\sigma}\overline{\psi}_A\gamma^{ab}\psi^A+\frac{1}{4}ge^{\sigma}\overline{\psi}_A\gamma^{ab}\psi^A\,,\\
H^{(\mathrm{new})} &= H\,,\\
F^{(\mathrm{new})} &= F-mB\,,\\
F^{r\ (\mathrm{new})} &= F^r\,,\\
\rho_A^{(\mathrm{new})} &= \rho_A^{(g=0)}-\frac{i}{4}me^{-3\sigma}\gamma_a\psi_AV^a-\frac{i}{4}ge^{\sigma}\gamma_a\psi_AV^a\,,\\
D\chi_A^{(\mathrm{new})} &= D\chi_A^{(g=0)}+\frac{3}{4}me^{-3\sigma}\psi_A-\frac{1}{4}ge^{\sigma}\psi_A\,,\\
d\sigma^{(\mathrm{new})} &= d\sigma\,.
\end{aligned}\tag{2.44}$$

It is now immediate to see that, in the vacuum defined by eq. (2.41), the dilatino field strength vanishes only for $g=3m$. Furthermore in this case, the extra g and m terms in the gravitino field strength are the correct ones in order to reconstruct the vanishing of the AdS covariant gravitino curvature while the extra term $-mB$ in $F^{(\mathrm{new})}$ and $4m^2\,\epsilon^{-6\sigma}V^aV^b+\frac{1}{4}me^{-3\sigma}\overline{\psi}_A\gamma^{ab}\psi^A$ in $R^{ab\,(\mathrm{new})}$ are exactly what is needed in order to have a supersymmetric AdS background (see eqs. (2.11)–(2.16)).

From the superspace solution of the Bianchi identities one immediately derives the space-time supersymmetry transformation law of the physical fields:

$$\begin{aligned}
\delta V^a_\mu &= -i\overline{\psi}^A_\mu\gamma^a\varepsilon^A\,,\\
\delta B_{\mu\nu} &= 4i\,e^{-2\sigma}\overline{\chi}_A\gamma_7\gamma_{\mu\nu}\varepsilon^A-4e^{-2\sigma}\overline{\varepsilon}_A\gamma_7\gamma_{[\mu}\psi^A_{\nu]}\,,\\
\delta A_\mu &= -2\,e^{\sigma}\overline{\chi}_A\gamma_7\gamma_\mu\varepsilon^A+2ie^{\sigma}\overline{\varepsilon}_A\gamma^7\psi^A_\mu\,,\\
\delta A^r_\mu &= 2e^{\sigma}\overline{\chi}^A\gamma_\mu\varepsilon^B\sigma^r_{AB}+2ie^{\sigma}\sigma^{rAB}\overline{\varepsilon}_A\psi_{B\mu}\,,\\
\delta\psi_{A\mu} &= D_\mu\varepsilon_A+\frac{1}{16}e^{-\sigma}[\epsilon_{AB}\widetilde{F}_{\nu\lambda}\gamma_7-\sigma_{rAB}\widetilde{F}^r_{\nu\lambda}](\gamma_\mu^{\ \nu\lambda}-6\delta^\nu_\mu\gamma^\lambda)\,\varepsilon^B+\\
&\quad+\frac{i}{32}e^{2\sigma}\widetilde{H}_{\nu\lambda\sigma}\;\gamma_7(\gamma_\mu^{\ \nu\lambda\sigma}-3\delta^\nu_\mu\gamma^{\lambda\sigma})\varepsilon_A-\frac{i}{4}g\;e^{\sigma}\gamma_\mu\varepsilon_A-\frac{i}{4}me^{-3\sigma}\gamma_\mu\varepsilon_A+\\
&\quad+\frac{1}{2}\gamma_7\varepsilon_A\overline{\chi}^C\gamma^7\psi_{\mu C}-\gamma_\nu\varepsilon_A\overline{\chi}^C\gamma^\nu\psi_{\mu C}+\gamma_7\gamma_\nu\varepsilon_A\overline{\chi}^C\gamma^7\gamma^\nu\psi_{\mu C}-\\
&\quad-\frac{1}{4}\gamma_{\nu\lambda}\varepsilon_A\overline{\chi}^C\gamma^{\nu\lambda}\psi_{\mu C}-\frac{1}{4}\gamma_7\gamma_{\nu\lambda}\varepsilon_A\overline{\chi}^C\gamma^7\gamma^{\nu\lambda}\psi_{\mu C}+\frac{1}{2}\varepsilon_A\overline{\chi}^C\psi_{\mu C}\,,
\end{aligned}\tag{2.45}$$

$$\begin{aligned}
\delta\chi_A &= \frac{i}{2}\gamma^\mu\widetilde{\partial_\mu\sigma}\varepsilon_A+\frac{i}{16}e^{\sigma}[\epsilon_{AB}\widetilde{F}_{\mu\nu}\gamma_7+\sigma_{rAB}\widetilde{F}^r_{\mu\nu}]\gamma^{\mu\nu}\varepsilon^B+\\
&\quad+\frac{1}{32}e^{2\sigma}\widetilde{H}_{\mu\nu\lambda}\gamma_7\gamma^{\mu\nu\lambda}\varepsilon_A-\frac{1}{4}ge^{\sigma}\varepsilon_A+\frac{3}{4}me^{-3\sigma}\varepsilon_A\,,
\end{aligned}\tag{2.46}$$

$$\delta\sigma = \overline{\chi}_A\varepsilon^A\,.$$

Apart from different conventions and normalizations the above equations coincide with the results of ref. [6], except for the extra terms in the gravitino transformation law of the form $\psi\chi\varepsilon$, which, like all the three-fermion terms, were not computed in ref. [6]. However these terms correspond to terms $\psi\psi\chi$ in the superspace curvature $D\psi_A$ which are quite essential to verify the consistency of the Bianchi identities when the parameter m is introduced and a supersymmetric AdS background is found; therefore they have an important meaning for the consistence of the theory and this is the reason why we have explicitly quoted them. This is to be contrasted with other three-fermion terms of the form $\chi\chi\varepsilon$ on space-time ($\chi\chi\psi$ in superspace), which we have not included in the transformation law of the fermions, since their explicit form can be found from the Bianchi identities once the consistency in the higher sectors has been verified, so that they are not on the same status. In the supersymmetric AdS vacuum we get:

$$\delta\chi_A = 0\,, \tag{2.47}$$

$$\delta\psi_{A\mu} = \nabla_\mu^{AdS}\epsilon_A\,, \tag{2.48}$$

$$R^{ab} \equiv -\frac{1}{2}R^{ab}_{cd}V^cV^d = -4m^2V^aV^b \longrightarrow R_{\mu\nu} = 20m^2 g_{\mu\nu}\,, \tag{2.49}$$

$$\mathcal{F}^r_{\mu\nu} = \mathcal{F}_{\mu\nu} - mB_{\mu\nu} = \chi_A = \psi_{A\mu} = \sigma = 0\,. \tag{2.50}$$

3. Coupling to matter multiplets

In $D = 6,\ N = 4$ supergravity, the only kind of matter is given by vector multiplets, namely

$$(A_\mu,\ \lambda_A,\ \phi^\alpha)^I\,, \tag{3.1}$$

where $\alpha = 0, 1, 2, 3$ and the index I labels an arbitrary number n of such multiplets. As it is well known the $4n$ scalars parametrize the coset manifold $\mathrm{SO}(4,n)/\,\mathrm{SO}(4) \times \mathrm{SO}(n)$. Taking into account that the pure supergravity has a non-compact duality group $O(1,1)$ parametrized by e^σ, the duality group of the matter coupled theory is

$$\frac{G}{H} = \frac{\mathrm{SO}(4,n)}{\mathrm{SO}(4)\times\mathrm{SO}(n)} \times O(1,1)\,. \tag{3.2}$$

To perform the matter coupling we follow the geometrical procedure of introducing the coset representative $L^\Lambda{}_\Sigma$ of the matter coset manifold, where $\Lambda, \Sigma, \ldots = 0, \ldots, 3 + n$; decomposing the $O(4,n)$ indices with respect to $H = \mathrm{SO}(4)\times \mathrm{O}(n)$ we have:

$$L^\Lambda{}_\Sigma = \left(L^\Lambda{}_\alpha, L^\Lambda{}_I\right), \tag{3.3}$$

where $\alpha = 0, 1, 2, 3,\ I = 4, \ldots, 3 + n$. Furthermore, since we are going to gauge the $\mathrm{SU}(2)$ diagonal subgroup of $\mathrm{O}(4)$ as in pure supergravity, we will also decompose $L^\Lambda{}_\alpha$ as

$$L^\Lambda{}_\alpha = (L^\Lambda{}_0, L^\Lambda{}_r)\,. \tag{3.4}$$

The $4+n$ gravitational and matter vectors will now transform in the fundamental of $\mathrm{SO}(4,n)$ so that the superspace vector curvatures will be now labeled by the index Λ: $F^\Lambda \equiv (F^0, F^r, F^I)$. Furthermore the covariant derivatives acting on the spinor fields will now contain also the composite connections of the $\mathrm{SO}(4,n)$ duality group. Let us introduce the left-invariant 1-form of $\mathrm{SO}(4,n)$

$$\Omega^\Lambda{}_\Sigma = (L^\Lambda{}_\Pi)^{-1} dL^\Pi{}_\Sigma \tag{3.5}$$

satisfying the Maurer-Cartan equation

$$d\Omega^\Lambda{}_\Sigma + \Omega^\Lambda{}_\Pi \wedge \Omega^\Pi{}_\Sigma = 0\,. \tag{3.6}$$

By appropriate decomposition of the indices, we find:

$$\begin{aligned}
R^r{}_s &= -P^r{}_I \wedge P^I{}_s\,,\\
R^r{}_0 &= -P^r{}_I \wedge P^I{}_0\,,\\
R^I{}_J &= -P^I{}_r \wedge P^r{}_J - P^I{}_0 \wedge P^0{}_J\,,\\
\nabla P^I_r &= 0\,,\\
\nabla P^I_0 &= 0\,,
\end{aligned} \tag{3.7}$$

where

$$\begin{aligned}
R^{rs} &\equiv d\Omega^r{}_s + \Omega^r{}_t \wedge \Omega^t{}_s + \Omega^r{}_0 \wedge \Omega^0{}_s\,,\\
R^{r0} &\equiv d\Omega^r{}_0 + \Omega^r{}_t \wedge \Omega^t{}_0\,,\\
R^{IJ} &\equiv d\Omega^I{}_J + \Omega^I{}_K \wedge \Omega^K{}_J
\end{aligned} \tag{3.8}$$

and we have set

$$P^I_\alpha = \begin{cases} P^I_0 \equiv \Omega^I{}_0 \\ P^I_r \equiv \Omega^I{}_r\,. \end{cases}$$

Note that P^I_0, P^I_r are the vielbeins of the coset, while $(\Omega^{rs},\ \Omega^{r0})$, $(R^{rs},\ R^{ro})$ are respectively the connections and the curvatures of $\mathrm{SO}(4)$ decomposed with respect to the diagonal subgroup $\mathrm{SU}(2) \subset \mathrm{SO}(4)$.

In terms of the previous definitions, the ungauged superspace curvatures of the matter coupled theory, generalizing eqs. (2.18)–(2.25) (with $m=0$) are now given by:

$$\begin{aligned}
T^A &= \mathcal{D}V^a - \frac{i}{2}\ \overline{\psi}_A \gamma_a \psi^A V^a = 0\,,\\
R^{ab} &= \mathcal{R}^{ab}\,,\\
H &= dB + 2e^{-2\sigma}\ \overline{\psi}_A \gamma_7 \gamma_a \psi^A V^a\,,\\
F^\Lambda &= \mathcal{F}^\Lambda - ie^\sigma L^\Lambda_0 \epsilon^{AB} \overline{\psi}_A \gamma_7 \psi_B - ie^\sigma L^\Lambda_r \sigma^{rAB} \overline{\psi}_A \psi_B\,,\\
\rho_A &= \mathcal{D}\psi_A - \frac{i}{2}\sigma_{rAB}\left(-\frac{1}{2}\epsilon^{rst}\Omega_{st} - i\gamma_7 \Omega_{r0}\right)\psi^B\,,
\end{aligned}$$

$$
\begin{aligned}
D\chi_A &= \mathcal{D}\chi_A - \frac{i}{2}\sigma_{rAB}\left(-\frac{1}{2}\epsilon^{rst}\Omega_{st} - i\gamma_7\Omega_{r0}\right)\chi^B\,,\\
R(\sigma) &= d\sigma\,,\\
\nabla\lambda_{IA} &= \mathcal{D}\lambda_{IA} - \frac{i}{2}\sigma_{rAB}\left(-\frac{1}{2}\epsilon^{rst}\Omega_{st} - i\gamma_7\Omega_{r0}\right)\lambda_I^B\,,\\
R_0^I(\phi) &\equiv P_0^I\,,\\
R_r^I(\phi) &\equiv P_r^I\,,
\end{aligned}
\tag{3.9}
$$

where the last two equations define the "curvatures" of the matter scalar fields ϕ^i as the vielbein of the coset:

$$
P_0^I \equiv P_{0i}^I d\phi^i\,, \qquad P_r^I \equiv P_{ri}^I d\phi^i\,,
\tag{3.10}
$$

where i runs over the $4n$ values of the coset vielbein world-components.

As in the pure supergravity case one can now write down the superspace Bianchi identities for the matter coupled curvatures. The computation is rather long but straightforward. We limit ourselves to give the new transformation laws of all the physical fields when matter is present, as derived from the solutions of the Bianchi identities.

$$
\begin{aligned}
\delta V_\mu^a &= -i\overline{\psi}_{A\mu}\gamma^a\varepsilon^A\,,\\
\delta B_{\mu\nu} &= 2e^{-2\sigma}\overline{\chi}_A\gamma_7\gamma_{\mu\nu}\varepsilon^A - 4e^{-2\sigma}\overline{\varepsilon}_A\gamma_7\gamma_{[\mu}\psi_{\nu]}^A\,,\\
\delta A_\mu^\Lambda &= 2e^\sigma\overline{\varepsilon}^A\gamma_7\gamma_\mu\chi^B L_0^\Lambda\epsilon_{AB} + 2e^\sigma\overline{\varepsilon}^A\gamma_\mu\chi^B L^{\Lambda r}\sigma_{rAB} - e^\sigma L_\Lambda^I\overline{\varepsilon}^A\gamma_\mu\lambda^{IB}\epsilon_{AB} +\\
&\quad + 2ie^\sigma L_0^\Lambda\overline{\varepsilon}_A\gamma^7\psi_B\epsilon^{AB} + 2ie^\sigma L^{\Lambda r}\sigma_r^{AB}\overline{\varepsilon}_A\psi_B\,,\\
\delta\psi_{A\mu} &= \mathcal{D}_\mu\varepsilon_A + \frac{1}{16}e^{-\sigma}[T_{[AB]\nu\lambda}\gamma_7 - T_{(AB)\nu\lambda}](\gamma_\mu{}^{\nu\lambda} - 6\delta_\mu^\nu\gamma^\lambda)\varepsilon^B +\\
&\quad + \frac{i}{32}e^{2\sigma}H_{\nu\lambda\rho}\gamma_7(\gamma_\mu{}^{\nu\lambda\rho} - 3\delta_\mu^\nu\gamma^{\lambda\rho})\varepsilon_A + \frac{1}{2}\varepsilon_A\overline{\chi}^C\psi_{C\mu} +\\
&\quad + \frac{1}{2}\gamma_7\varepsilon_A\overline{\chi}^C\gamma^7\psi_{C\mu} - \gamma_\nu\varepsilon_A\overline{\chi}^C\gamma^\nu\psi_{C\mu} + \gamma_7\gamma_\nu\varepsilon_A\overline{\chi}^C\gamma^7\gamma^\nu\psi_{C\mu} -\\
&\quad - \frac{1}{4}\gamma_{\nu\lambda}\varepsilon_A\overline{\chi}^C\gamma^{\nu\lambda}\psi_{C\mu} - \frac{1}{4}\gamma_7\gamma_{\nu\lambda}\varepsilon_A\overline{\chi}^C\gamma^7\gamma^{\nu\lambda}\psi_{C\mu}\,,
\end{aligned}
\tag{3.11}
$$

$$
\begin{aligned}
\delta\chi_A &= \frac{i}{2}\gamma^\mu\partial_\mu\sigma\varepsilon_A + \frac{i}{16}e^{-\sigma}[T_{[AB]\mu\nu}\gamma_7 + T_{(AB)\mu\nu}]\gamma^{\mu\nu}\varepsilon^B +\\
&\quad + \frac{1}{32}e^{2\sigma}H_{\mu\nu\lambda}\gamma_7\gamma^{\mu\nu\lambda}\varepsilon_A\,,
\end{aligned}
\tag{3.12}
$$

$$
\begin{aligned}
\delta\sigma &= \overline{\chi}_A\varepsilon^A\,,\\
\delta\lambda^{IA} &= -iP_{ri}^I\sigma^{rAB}\partial_\mu\phi^i\gamma^\mu\varepsilon_B + iP_{0i}^I\epsilon^{AB}\partial_\mu\phi^i\gamma^7\gamma^\mu\varepsilon_B + \frac{i}{2}e^{-\sigma}T_{\mu\nu}^I\gamma^{\mu\nu}\varepsilon^A\,,
\end{aligned}
\tag{3.13}
$$

$$
\begin{aligned}
P_{0i}^I\delta\phi^i &= \frac{1}{2}\overline{\lambda}_A^I\gamma_7\varepsilon^A\,,\\
P_{ri}^I\delta\phi^i &= \frac{1}{2}\overline{\lambda}_A^I\varepsilon_B\sigma_r^{AB}\,,
\end{aligned}
$$

where we have introduced the "dressed" vector field strengths

$$\begin{aligned}
T_{[AB]\mu\nu} &\equiv \epsilon_{AB} L^{-1}_{0\Lambda} F^{\Lambda}_{\mu\nu}\,, \\
T_{(AB)\mu\nu} &\equiv \sigma^{r}_{AB} L^{-1}_{r\Lambda} F^{\Lambda}_{\mu\nu}\,, \\
T_{I\mu\nu} &\equiv L^{-1}_{I\Lambda} F^{\Lambda}_{\mu\nu}
\end{aligned} \tag{3.14}$$

and we have omitted in the transformation laws of the fermions the three-fermions terms of the form $(\chi\chi\varepsilon)$, $(\lambda\lambda\varepsilon)$.

We observe that the solutions of the Bianchi identities also imply the equations of motion of the physical fields and therefore one can reconstruct in principle the space-time lagrangian. Nevertheless, in general, it is simpler to construct the lagrangian explicitly and we will present its complete expression in the forthcoming paper of ref. [16]. In the next section, however, we will need at least the kinetic terms and mass matrices terms in order to construct the scalar potential of the gauged theory. This is the topic of the next paragraph.

4. The gauging

The next problem we have to cope with is the gauging of the matter coupled theory and the determination of the scalar potential.

Let us first consider the ordinary gauging, with $m = 0$, which, as usual, will imply the presence of new terms proportional to the coupling constants in the supersymmetry transformation laws of the fermion fields.

Our aim is to gauge a compact subgroup of $O(4,n)$. Since in any case we may gauge only the diagonal subgroup $\mathrm{SU}(2) \subset \mathrm{O}(4) \subset H$, the maximal gauging is given by $\mathrm{SU}(2) \otimes \mathcal{G}$ where $\mathcal{G}$ is a n-dimensional subgroup of $\mathrm{O}(n)$. According to a well-known procedure, we modify the definition of the left invariant 1-form $L^{-1}dL$ by replacing the ordinary differential with the $\mathrm{SU}(2)\otimes\mathcal{G}$ covariant differential as follows:

$$\nabla L^{\Lambda}{}_{\Sigma} = dL^{\Lambda}{}_{\Sigma} - f_{\Gamma}{}^{\Lambda}{}_{\Pi} A^{\Gamma} L^{\Pi}{}_{\Sigma}\,, \tag{4.1}$$

where $f^{\Lambda}{}_{\Pi\Gamma}$ are the structure constants of $\mathrm{SU}(2)_d\otimes\mathcal{G}$. More explicitly, denoting with ϵ^{rst} and $\mathcal{C}^{IJK}$ the structure constants of the two factors $\mathrm{SU}(2)$ and $\mathcal{G}$, eq. (4.1) splits as follows:

$$\begin{aligned}
\nabla L^{0}{}_{\Sigma} &= dL^{\Lambda}{}_{\Sigma}\,, \\
\nabla L^{r}{}_{\Sigma} &= dL^{r}{}_{\Sigma} - g\epsilon_{t}{}^{r}{}_{s} A^{t} L^{s}{}_{\Sigma}\,, \\
\nabla L^{I}{}_{\Sigma} &= dL^{I}{}_{\Sigma} - g'\mathcal{C}_{K}{}^{I}{}_{J} A^{K} L^{J}{}_{\Sigma}\,.
\end{aligned} \tag{4.2}$$

Setting $\widehat{\Omega} = L^{-1}\nabla L$, one easily obtains the gauged Maurer-Cartan equations:

$$d\widehat{\Omega}^{\Lambda}{}_{\Sigma} + \widehat{\Omega}^{\Lambda}{}_{\Pi} \wedge \widehat{\Omega}^{\Pi}{}_{\Sigma} = (L^{-1}\mathcal{F}L)^{\Lambda}{}_{\Sigma}\,, \tag{4.3}$$

where $\mathcal{F} \equiv \mathcal{F}^{\Lambda} T_{\Lambda}$, T_{Λ} being the generators of $\mathrm{SU}(2)\otimes\mathcal{G}$.

After gauging, the same decomposition as in eqs. (3.7) now gives:

$$
\begin{aligned}
R^r{}_s &= -P^r{}_I \wedge P^I{}_s + (L^{-1}\mathcal{F}L)^r{}_s \,, \\
R^r{}_0 &= -P^r{}_I \wedge P^I{}_0 + (L^{-1}\mathcal{F}L)^r{}_0 \,, \\
R^I{}_J &= -P^I{}_r \wedge P^r{}_J - P^I{}_0 \wedge P^0{}_J + (L^{-1}\mathcal{F}L)^I{}_J \,, \\
\nabla P^I_r &= (L^{-1}\mathcal{F}L)^I{}_r \,, \\
\nabla P^I_0 &= (L^{-1}\mathcal{F}L)^I{}_0 \,.
\end{aligned} \tag{4.4}
$$

Because of the presence of the gauged terms in the coset curvatures, the new Bianchi Identities are not satisfied by the old superspace curvatures but we need extra terms in the fermion field strengths parametrizations, that is, in space-time language, extra terms in the transformation laws of the fermion fields of eqs. (3.11), (3.12) and (3.13), named "fermionic shifts".

$$\delta\psi_{A\mu} = \delta\psi^{(\mathrm{old})}_{A\mu} + S_{AB}(g,g')\gamma_\mu\varepsilon^B \,, \tag{4.5}$$

$$\delta\chi_A = \delta\chi^{(\mathrm{old})}_A + N_{AB}(g,g')\varepsilon^B \,, \tag{4.6}$$

$$\delta\lambda^I_A = \delta\lambda^{I(\mathrm{old})}_A + M^I_{AB}(g,g')\varepsilon^B \,. \tag{4.7}$$

Again, working out the Bianchi identities, one fixes the explicit form of the fermionic shifts which turn out to be

$$S^{(g,g')}_{AB} = \frac{i}{24}Ae^\sigma\epsilon_{AB} - \frac{i}{8}B_t\gamma^7\sigma^t_{AB} \,, \tag{4.8}$$

$$N^{(g,g')}_{AB} = \frac{1}{24}Ae^\sigma\epsilon_{AB} + \frac{1}{8}B_t\gamma^7\sigma^t_{AB} \,, \tag{4.9}$$

$$M^{I(g,g')}_{AB} = (-C^I_t + 2i\gamma^7 D^I_t)\sigma^t_{AB} \,, \tag{4.10}$$

where

$$A = \epsilon^{rst}K_{rst} \,, \tag{4.11}$$

$$B^i = \epsilon^{ijk}K_{jk0} \,, \tag{4.12}$$

$$C^t_I = \epsilon^{trs}K_{rIs} \,, \tag{4.13}$$

$$D_{It} = K_{0It} \tag{4.14}$$

and the threefold completely antisymmetric tensors $K's$ are the so called "boosted structure constants" given explicitly by:

$$
\begin{aligned}
K_{rst} &= g\epsilon_{lmn}L^l{}_r(L^{-1})_s{}^m L^n{}_t + g'\mathcal{C}_{IJK}L^I{}_r(L^{-1})_s{}^J L^K_t \,, \\
K_{rs0} &= g\epsilon_{lmn}L^l{}_r(L^{-1})_s{}^m L^n_0 + g'\mathcal{C}_{IJK}L^I{}_r(L^{-1})_s{}^J L^K_0 \\
K_{rIt} &= g\epsilon_{lmn}L^l{}_r(L^{-1})_I{}^m L^n{}_t + g'\mathcal{C}_{LJK}L^L{}_r(L^{-1})_I{}^J L^K_t \,, \\
K_{0It} &= g\epsilon_{lmn}L^l{}_0(L^{-1})_I{}^m L^n{}_t + g'\mathcal{C}_{LJK}L^L_0(L^{-1})_I{}^J L^K_t \,.
\end{aligned} \tag{4.15}
$$

Actually one easily see that the fermionic shifts (4.8) and (4.9) reduce to the pure supergravity g dependent terms of eqs. (2.38) and (2.39). (Note that, since $L^\Lambda{}_\Sigma \to \delta^\Lambda_\Sigma$ in absence of matter, the terms proportional to the Pauli σ matrices are simply absent in such a limit.)

At this point one could compute the scalar potential of the matter coupled theory, in terms of the fermionic shifts just determined, using the well-known Ward identity of the scalar potential which can be derived from the lagrangian. Since we are going to perform this derivation once we will introduce also m dependent terms in the fermionic shifts, we just quote for the moment, the expected result that the potential due only to g and g' dependent shifts does not admit a stable AdS background configuration. We are thus, led as in the pure supergravity case, to determine suitable m dependent terms that reduce to the m terms of eqs. (2.45) and (2.46) in absence of matter multiplets. One can see that a simple-minded ansatz of keeping exactly the same form for the m dependent terms as in the pure supergravity case is not consistent with the gauged superspace Bianchi identities.

It turns out that a consistent solution for the relevant m terms to be added to the fermionic shifts, implies the presence of the coset representatives; that is, the m-terms must also be "dressed" with matter scalar fields as it happens for the g and g' dependent terms. Explicitly, the Bianchi identities solution for the new fermionic shifts is:

$$\begin{aligned}
S^{(g,g',m)}_{AB} &= \frac{i}{24}\left[Ae^\sigma + 6me^{-3\sigma}(L^{-1})_{00}\right]\epsilon_{AB} - \\
&\quad - \frac{i}{8}\left[B_t e^\sigma - 2me^{-3\sigma}(L^{-1})_{i0}\right]\gamma^7\sigma^t_{AB}\,, && (4.16)\\
N^{(g,g',m)}_{AB} &= \frac{1}{24}\left[Ae^\sigma - 18me^{-3\sigma}(L^{-1})_{00})\right]\epsilon_{AB} + \\
&\quad + \frac{1}{8}\left[B_t e^\sigma + 6me^{-3\sigma}(L^{-1})_{i0}\right]\gamma^7\sigma^t_{AB}\,, && (4.17)\\
M^{I(g,g',m)}_{AB} &= \left(-C^I_t + 2i\gamma^7 D^I_t\right)e^\sigma\sigma^t_{AB} - 2me^{-3\sigma}(L^{-1})^I{}_0\gamma^7\epsilon_{AB}\,. && (4.18)
\end{aligned}$$

Let us note that, similarly to what happens in the pure supergravity case, the m dependent terms behave in an analogous way as the g dependent gauging terms, the former being dressed by the scalar fields in a similar way as the latters. The difference is that, while the gauged terms are threelinear in the coset representatives of the duality group, the m terms are linear (recall that because of the pseudo-orthogonality condition, $L^{-1} = \eta L^T \eta$).[3] Furthermore we note that, as it happens for the g and g' terms, the matter coupling forces new terms in the fermionic shifts proportional to σ^t_{AB} while for the gaugino shift the m terms contribute terms proportional to ϵ_{AB}.

[3] Amusingly enough the reverse situation happens for the "coset" representative of $O(1,1)$: the g terms are linear in e^σ, while the m terms are threelinear, being proportional to $e^{-3\sigma}$.

5. The scalar potential

The simplest way to derive the scalar potential is to use the supersymmetry Ward identity which relates the scalar potential to the fermionic shifts in the transformation laws [18]. In order to retrieve such identity it is necessary to have the relevant terms of the lagrangian of the gauged theory. These terms are actually the kinetic ones and the "mass" terms given in the following equation:

$$\begin{aligned}(\det V)^{-1}\mathcal{L} = & -\frac{1}{4}\mathcal{R} - \frac{1}{8}e^{2\sigma}\mathcal{N}_{\Lambda\Sigma}\widehat{\mathcal{F}}^{\Lambda}_{\mu\nu}\widehat{\mathcal{F}}^{\Sigma\mu\nu} + \partial^{\mu}\sigma\partial_{\mu}\sigma - \frac{1}{4}(P^{I0}_{\mu}P^{\mu}_{I0} + P^{Ir}_{\mu}P^{\mu}_{Ir}) - \\ & - \frac{i}{2}\overline{\psi}_{A\mu}\gamma^{\mu\nu\rho}D_{\nu}\psi^{A}_{\rho} + \frac{i}{8}\overline{\lambda}^{I}_{A}\gamma^{\mu}D_{\mu}\lambda^{A}_{I} - 2i\overline{\chi}_{A}\gamma^{\mu}D_{\mu}\chi^{A} + 2i\overline{\psi}^{A}_{\mu}\gamma^{\mu\nu}\overline{S}_{AB}\psi^{B}_{\nu} + \\ & + 4i\overline{\psi}^{A}_{\mu}\gamma^{\mu}\overline{N}_{AB}\chi^{B} + \frac{i}{4}\overline{\psi}^{A}_{\mu}\gamma^{\mu}\overline{M}^{I}_{AB}\lambda^{B}_{I} + \mathcal{W}(\sigma\phi^{i}; g, g', m) + \cdots , \end{aligned} \tag{5.1}$$

where

$$\mathcal{N}_{\Lambda\Sigma} = L_{\Lambda}^{\ 0}L^{-1}_{0\Sigma} + L_{\Lambda}^{\ i}L^{-1}_{i\Sigma} - L_{\Lambda}^{\ I}L^{-1}_{I\Sigma} \tag{5.2}$$

is the vector kinetic matrix, $\widehat{\mathcal{F}}^{\Lambda}_{\mu\nu} \equiv \mathcal{F}^{\Lambda}_{\mu\nu} - m\delta^{\Lambda}_{0}B_{\mu\nu}$ and $\mathcal{W}$ is minus the scalar potential.

In eq. (5.1) there appear "barred mass-matrices" $\overline{S}_{AB}$, $\overline{N}_{AB}$, $\overline{M}^{I}_{AB}$ which are slightly different from the fermionic shifts defined in eqs. (4.16), (4.17) and (4.18). Actually they are defined by:

$$\overline{S}_{AB} = -S_{BA}\,, \qquad \overline{N}_{AB} = -N_{BA}\,, \qquad \overline{M}^{I}_{AB} = M^{I}_{BA}\,. \tag{5.3}$$

Definitions (5.3) stem from the fact that the shifts defined in eqs. (4.16), (4.17) and (4.18) are matrices in the eight-dimensional spinor space, since they contain the γ_7 matrix; as will be seen in a moment, such definition is actually necessary in order to satisfy the supersymmetry Ward identity.

Indeed, let us perform the supersymmetry variation of (5.1), keeping only the terms proportional to g, g' or m, and to the current $\overline{\psi}_{A\mu}\gamma^{\mu}\epsilon^{A}$; we find the following Ward identity:

$$\delta^{C}_{A}\mathcal{W} = 20\overline{S}^{AB}S_{BC} + 4\overline{N}^{AB}N_{BC} + \frac{1}{4}\overline{M}^{AB}_{I}M^{I}_{BC}\,. \tag{5.4}$$

However we note that, performing the supersymmetry variation, the gauge terms also give rise to extra terms proportional to the current $\overline{\psi}_{A\mu}\gamma^{7}\gamma^{\mu}\epsilon^{A}$, which have no counterpart in the term containing the potential $\mathcal{W}$. Because of the definition of the barred mass matrices in eq. (5.3) it is easily seen that such "γ^7-terms", arising from $\overline{S}^{AB}S_{BC}$ and $\overline{N}^{AB}N_{BC}$ cancel against each other.

As far as the term $\overline{M}^{AB}_{I}M^{I}_{BC}$ is concerned, the same mechanism of cancellation again applies to the terms proportional to $\overline{\psi}_{A\mu}\gamma_7\gamma^{\mu}\epsilon^{C}\sigma^{rA}_{C}$; there is, however, a residual dangerous term of the form

$$\delta^{A}_{C}\overline{\psi}_{A\mu}\gamma^{\mu}\gamma^{7}D^{I}_{\ s}C_{I}^{\ s}\epsilon^{C}\,. \tag{5.5}$$

One can show that this term vanishes identically owing to the non trivial relation

$$D^I_t C^t_I = 0\,. \tag{5.6}$$

Equation (5.6) can be shown to hold using the pseudo-orthogonality relation $L^T\eta\, L = \eta$ among the coset representatives and the Jacobi identities $C_{I[JK}C_{L]MN} = 0$, $\epsilon_{r[st}\epsilon_{l]mn} = 0$. This is a non-trivial check of our computation.

It now follows that the Ward identity eq. (5.4) is indeed satisfied since all the terms on the r.h.s., once the "γ^7-terms" have been cancelled, are proportional to δ^C_A.

Using the expressions (4.16), (4.17), (4.18) and (5.3) in eq. (5.4), we obtain the explicit form of the scalar potential

$$\begin{aligned}\mathcal{W}(\phi) = 5&\left\{\left[\frac{1}{12}(Ae^{\sigma}+6me^{-3\sigma}L_{00})\right]^2+\left[\frac{1}{4}(e^{\sigma}B_i-2me^{-3\sigma}L_{0i})\right]^2\right\}-\\ &-\left\{\left[\frac{1}{12}(Ae^{\sigma}-18me^{-3\sigma}L_{00})\right]^2+\left[\frac{1}{4}(e^{\sigma}B_i+6me^{-3\sigma}L_{0i})\right]^2\right\}-\\ &-\frac{1}{4}\left\{C^I{}_tC_{It}+4D^I{}_tD_{It}\right\}\ e^{2\sigma}-m^2e^{-6\sigma}L_{0I}L^{0I}\,.\end{aligned} \tag{5.7}$$

Let us note that, setting

$$\mathcal{H} = \frac{1}{12}\left(Ae^{\sigma}+6me^{-3\sigma}L_{00}\right), \tag{5.8}$$

$$\mathcal{K}_i = \frac{1}{4}\left(e^{\sigma}B_i+6me^{-3\sigma}L_{0i}\right) \tag{5.9}$$

the potential can be written as follows:

$$\begin{aligned}\mathcal{W} = 5\{\mathcal{H}^2+\mathcal{K}^i\mathcal{K}_i\}-\left\{[\partial_\sigma\mathcal{H}]^2+\partial_\sigma\mathcal{K}^i\nabla_\sigma\mathcal{K}_i\right\}-\\ -2\left\{\nabla_{I\alpha}\mathcal{H}\nabla^{I\alpha}\mathcal{H}+\nabla_{I\alpha}\mathcal{K}_i\nabla^{I\alpha}\mathcal{K}^i\right\}_{m=0}-\left\{\nabla_{I\alpha}\mathcal{H}\nabla^{I\alpha}\mathcal{H}+\nabla_{I\alpha}\mathcal{K}_i\nabla^{I\alpha}\mathcal{K}^i\right\}_{g=0}\end{aligned} \tag{5.10}$$

or alternatively

$$\begin{aligned}\mathcal{W} = 5\{\mathcal{H}^2+\mathcal{K}^i\mathcal{K}_i\}-\left\{[\partial_\sigma\mathcal{H}]^2+\partial_\sigma\mathcal{K}^i\partial_\sigma\mathcal{K}_i\right\}-\\ -2\left\{\nabla_{I\alpha}\mathcal{H}\nabla^{I\alpha}\mathcal{H}+\nabla_{I\alpha}\mathcal{K}_i\nabla^{I\alpha}\mathcal{K}^i\right\}+m^2e^{-6\sigma}L_{0I}L^{0I}\,,\end{aligned} \tag{5.11}$$

where $\nabla_{I\alpha} \equiv (\nabla_{I0}, \nabla_{Ir})$ denote the derivatives with respect to the "linearized coordinates": that is, using the Maurer-Cartan equations

$$\nabla^H L^\Lambda{}_I = L^\Lambda{}_\alpha P^\alpha_I\,, \qquad \nabla^H L^\Lambda{}_\alpha = L^\Lambda{}_I P^I_\alpha \tag{5.12}$$

the flat derivative $\nabla_{I\alpha}$ are defined as the coefficient of the coset vielbein $P^{I\alpha}$ in eqs. (5.12). In deriving equations (5.10) and (5.11) one has to make use of the following relations which are a straightforward consequence of the definitions (4.11)–(4.14)

$$\begin{aligned} \nabla_{I0} A &= 0\,, \\ \nabla_{Ir} A &= 3C_{Ir}\,, \\ \nabla_{I0} B_i &= C_{iI}\,, \\ \nabla_{Ir} B_i &= -2\epsilon_{rik} D_{Ik}\,. \end{aligned} \tag{5.13}$$

Expanding the squares in eq. (5.7) the potential $\mathcal{W}$ can be alternatively written as follows:

$$\mathcal{W} = e^{2\sigma}\left[\frac{1}{36}A^2 + \frac{1}{4}B^i B_i - \frac{1}{4}\left(C^I{}_t C_{It} + 4D^I{}_t D_{It}\right)\right] - m^2 e^{-6\sigma}\mathcal{N}_{00} + \\ + m e^{-2\sigma}\left[\frac{2}{3}AL_{00} - 2B^i L_{0i}\right], \tag{5.14}$$

where $\mathcal{N}_{00}$ is the (00) component of the vector kinetic matrix defined in eq. (5.2).

We now show that, apart from other possible extrema not considered here, a stable supersymmetric extremum of the potential $\mathcal{W}$ is found to be the same as in the case of pure supergravity, that is we get an AdS supersymmetric background only for $g = 3m$. In fact, setting $\partial_\sigma \mathcal{W} = 0$ and keeping only the non-vanishing terms at $\sigma = q^{I\alpha} = 0$, $q^{I\alpha}$ being the flat coordinates, we have

$$\partial_\sigma \mathcal{W} = \left[\frac{1}{18}A^2 e^{2\sigma} - \frac{4}{3}mAL_{00}e^{-2\sigma} + 6m^2 L_{00}^2 e^{-6\sigma}\right]_{\sigma = q^{I\alpha} = 0} \tag{5.15}$$

since all the other terms entering the $\partial_\sigma \mathcal{W}$ contain at least one off-diagonal element of the coset representative which vanishes identically when the scalar fields are set equal to zero. Furthermore, from the definition (4.11) and using $L^\Lambda{}_\Sigma(q^I_\alpha = 0) = \delta^\Lambda_\Sigma$, we find:

$$A(q^I_\alpha = 0) = 6g\,; \qquad L_{00}(q^I_\alpha = 0) = 1 \tag{5.16}$$

so that

$$\partial_\sigma \mathcal{W}|_{\sigma = q = 0} = 2g^2 - 8mg + 6m^2 = 0\,. \tag{5.17}$$

As the partial derivatives $(\partial\mathcal{W}?\partial q^{I0})_{\sigma=q=0}$, $(\partial\mathcal{W}/\partial q^{Ir})_{\sigma=q=0}$ are also zero, since they contains at least one off-diagonal coset representative, the condition for the minimum is given by eq. (5.17) which coincides with the equation one obtains for the pure supergravity case, whose solutions are $g = m$, $g = 3m$.

Using eqs. (5.16), (5.17) and (4.5)–(4.7) (with the new shifts (4.16)–(4.18)) one can easily recognize that only the $g = 3m$ solution gives rise to a supersymmetric AdS background.

A further issue related to the scalar potential, which is an important check of all our calculation, is the possibility of computing the masses of the scalar fields by varying the linearized kinetic terms and the potential of (5.1), after power expansion of $\mathcal{W}$ up to the second order in the scalar fields q^I_α.

We find:

$$\left(\frac{\partial^2 \mathcal{W}}{\partial \sigma^2}\right)_{\sigma=q=0,g=3m} = 48m^2\,,$$
$$\left(\frac{\partial^2 \mathcal{W}}{\partial q^{I0}\partial q^{J0}}\right)_{\sigma=q=0,g=3m} = 8m^2\delta^{IJ}\,,$$
$$\left(\frac{\partial^2 \mathcal{W}}{\partial q^{Ir}\partial q^{Js}}\right)_{\sigma=q=0,g=3m} = 24m^2\delta^{IJ}\delta^{rs}\,. \tag{5.18}$$

The linearized equations of motion become:

$$\Box\sigma - 24m^2\sigma = 0\,,$$
$$\Box q^{I0} - 16m^2 q^{I0} = 0\,,$$
$$\Box q^{Ir} - 24m^2 q^{Ir} = 0\,. \tag{5.19}$$

If we use as mass unity the inverse AdS radius, which in our conventions (see eq. (2.49)) is $R_{\mathrm{AdS}}^{-2} = 4m^2$ we get:

$$m_\sigma^2 = -6\,, \qquad m_{q^{I0}}^2 = -4\,, \qquad m_{q^{Ir}}^2 = -6\,. \tag{5.20}$$

These values should be compared with the results obtained in ref. [5] where the supergravity and matter multiplets of the AdS_6 $F(4)$ theory were constructed in terms of the singleton fields of the 5-dimensional conformal field theory, the singleton being given by hypermultiplets transforming in the fundamental of $\mathcal{G} \equiv E_7$. It is amusing to see that the values of the masses of the scalars computed in terms of the conformal dimensions are exactly the same as those given in eq. (5.20).

This coincidence can be considered as a non-trivial check of the AdS/CFT correspondence in six versus five dimensions.

To make contact with what follows we observe that the scalar squares masses in AdS_{d+1} are given by the $\mathrm{SO}(2,d)$ quadratic Casimir [22]

$$m^2 = E_0(E_0 - d)\,. \tag{5.21}$$

They are negative in the interval $\frac{d-2}{2} \leq E_0 < d$ (the lower bound corresponding to the unitarity bound, i.e. the singleton) and attain the Breitenlohner-Freedman bound [17] when $E_0 = d - E_0$, i.e. at $E_0 = d/2$ for which $m^2 = -d^2/4$. Conformal propagation correspond to $m^2 = -\frac{d^2-1}{4}$, i.e. $E_0 = \frac{d\pm1}{2}$. This is the case of the dilaton and triplet matter scalars.

6. $F(4) \otimes \mathcal{G}$ superconformal field theory

In this section we describe the basics of the $F(4)$ highest weight unitary irreducible representations "UIR's" and exhibit two towers of short representations which are relevant for a K-K analysis of type-IIA theory on (warped) $AdS_6 \otimes S^4$ [8, 9].

We will not consider here the $\mathcal{G}$ representation properties but we will only concentrate on the supersymmetric structure.

Recalling that the even part of the $F(4)$ superalgebra is $\mathrm{SO}(2,5) \otimes \mathrm{SU}(2)$, from a general result on Harish-Chandra modules [19, 12, 20, 21] of $\mathrm{SO}(2, 2n+1)$ we know that there are only a spin 0 and a spin 1/2 singleton unitary irreducible representations [22], which, therefore, merge into a unique supersingleton representation of the $F(4)$ superalgebra: the hypermultiplet [5].

To describe shortening is useful to use a harmonic superfield language [13].

The harmonic space is in this case the 2-sphere[4] $\mathrm{SU}(2)/\mathrm{U}(1)$, as in $N = 2,\ d = 4$ and $N = 1,\ d = 6$. A highest weight UIR of $\mathrm{SO}(2,5)$ is determined by E_0 and a UIR of $\mathrm{SO}(5) \simeq \mathrm{USp}(4)$, with Dynkin labels $(a_1,\ a_2)$.[5] We will denote such representations by $\mathcal{D}(E_0,\ a_1,\ a_2)$. The two singletons correspond to $E_0 = 3/2,\ a_1 = a_2 = 0$ and $E_0 = 2,\ a_1 = 1,\ a_2 = 0$.

In the AdS/CFT correspondence $(E_0,\ a_1,\ a_2)$ become the conformal dimension and the Dynkin labels of $\mathrm{SO}(1,4) \simeq \mathrm{USp}(2,2)$.

The highest weight UIR of the $F(4)$ superalgebra will be denoted by $\mathcal{D}(E_0, a_1, a_2; I)$ where I is the $\mathrm{SU}(2)$ R-symmetry quantum number (integer or half integer).

We will show shortly that there are two (isolated) series of UIR's which correspond respectively to 1/2 BPS short multiplets (analytic superfields) and intermediate short superfields. The former have the property that they form a ring under multiplication, as the chiral fields in $d = 4$ [23].

The first series is the massive tower of short vector multiplets whose lowest members is a massless vector multiplet in $Adj\mathcal{G}$ corresponding to the conserved currents of the $\mathcal{G}$ global symmetry of the five-dimensional conformal field theory.

The other series is the tower of massive graviton multiplets, which exhibit "intermediate shortening" and it is not of BPS type. Its lowest member is the supergravity multiplet which contains the $\mathrm{SU}(2)$ R-symmetry current and the stress-tensor among the superfield components.

6.1 $F(4)$ superfields

The basic superfield is the supersingleton hypermultiplet $W^A(x,\theta)$, which satisfies the constraint

$$D_\alpha^{(A} W^{B)}(x,\theta) = 0\,, \tag{6.1}$$

corresponding to the irrep. $\mathcal{D}(E_0 = 3/2, 0, 0; I = 1/2)$ [13].

By using harmonic superspace, $(x,\ \theta_I,\ u_i^I)$, where $\theta_I = \theta_A u_I^A$, u_I^A is the coset representative of $\mathrm{SU}(2)/\mathrm{U}(1)$ and I is the charge U(1)-label, from the covariant

[4]The sphere is the simplest example of "flag manifold" whose geometric structure underlies the construction of harmonic superspaces [24]

[5]Note that the USp(4) Young labels h_1, h_2 are related to a_1, a_2 by $a_1 = 2h_2; a_2 = h_1 - h_2$.

derivative algebra

$$\{D^A_\alpha, D^B B_\beta\} = i\epsilon^{AB}\partial_{\alpha\beta} \tag{6.2}$$

we have

$$\{D^I_\alpha, D^I_\beta\} = 0\,, \qquad D^I_\alpha = D^A_\alpha u^I_A\,. \tag{6.3}$$

Therefore from eq. (6.1) it follows the G-analytic constraint:

$$D^1_\alpha W^1 = 0 \tag{6.4}$$

which implies

$$W^1(x,\theta) = \varphi^1 + \theta^\alpha_2 \zeta_\alpha + \text{d.t.} \tag{6.5}$$

(d.t. means "derivative terms").

Note that W^1 also satisfies

$$D^2_\alpha D^{2\alpha} W^1 = 0 \tag{6.6}$$

because there is no such scalar component[6] in W^1.

W^1 is a Grassman analytic superfield, which is also harmonic (that is $\mathbf{D}^1_2 W^1 = 0$ where, using notations of ref. [23], $\mathbf{D}^1_2$ is the step-up operator of the SU(2) algebra acting on harmonic superspace).

Since W^1 satifies $D^1 W^1 = 0$, any p-order polynomial

$$I_p(W^1) = (W^1)^p \tag{6.7}$$

will also have the same property, so these operators form a ring under multiplication [23], they are the 1/2 BPS states of the $F(4)$ superalgebra and represent massive vector multiplets ($p > 2$), and massless bulk gauge fields for $p = 2$.

The above multiplets correspond to the $D(E_0 = 3I, 0, 0; I = p/2)$ h.w. U.I.R.'s of the $F(4)$ superalgebra.

Note also that if W^1 carries a pseudo-real representation of the flavor group $\mathcal{G}$ (e.g. **56** of $\mathcal{G} = E_7$) then W^1 satisfies a reality condition

$$(W^1)^* = W^2 \tag{6.8}$$

corresponding to the superfield constraint

$$(W^A)^{*\Lambda} = \epsilon_{AB}\Omega_{\Lambda\Sigma} W^{jB\Sigma}\,. \tag{6.9}$$

The SU(2) quantum numbers of the W^{1p} superfield components are:

$$\begin{array}{lll} (\theta)^0 & \text{spin } 0 & I = \dfrac{p}{2} \\ (\theta)^1 & \text{spin } \dfrac{1}{2} & I = \dfrac{p}{2} - \dfrac{1}{2} \end{array}$$

[6]This is rather similar to the treatment of the (1,0) hypermultiplet in $D = 6$ [25]

$$
\begin{array}{ccc}
(\theta)^2 & \text{spin } 0 - \text{spin } 1 & I = \dfrac{p}{2} - 1 \\
(\theta)^3 & \text{spin } \dfrac{1}{2} & I = \dfrac{p}{2} - \dfrac{3}{2} \\
(\theta)^4 & \text{spin } 0 & I = \dfrac{p}{2} - 2
\end{array}
$$

Note that the $(\theta)^4$ component is missing for $p = 2$, $p = 3$, while the $(\theta)^3$ component is missing for $p = 2$. However the total number of states is $8(p-1)$ both for boson and fermion fields ($p \geq 2$).

The AdS squared mass for scalars is

$$
m_s^2 = E_0(E_0 - 5) \tag{6.10}
$$

so there are three families of scalar states with

$$
\begin{aligned}
m_1^2 &= \frac{3}{4}p(3p - 10)\,, & p \geq 2\,, \\
m_2^2 &= \frac{1}{4}(3p + 2)(3p - 8)\,, & p \geq 2\,, \\
m_3^2 &= \frac{1}{4}(3p + 4)(3p - 6)\,, & p \geq 4\,.
\end{aligned}
$$

The only scalars states with $m^2 < 0$ are the scalar in the massless vector multiplet ($p = 2$) with $m_1^2 = -6$, $m_2^2 = -4$ (no states with $m^2 = 0$ exist) and in the $p = 3$ multiplet with $m^2 = -9/4$.

We now consider the second "short" tower containing the graviton supermultiplet and its recurrences.

The graviton multiplet is given by $W^1\overline{W}^1$. Note that such superfield is not G-analytic, but it satisfies

$$
D_\alpha^1 D^{1\alpha}(W^1\overline{W}^1) = D_\alpha^2 D^{2\alpha}(W^1\overline{W}^1) = 0 \tag{6.11}
$$

this multiplet is the $F(4)$ supergravity multiplet. Its lowest component, corresponding to the dilaton in AdS_6 supergravity multiplet, is a scalar with $E_0 = 3$ ($m^2 = -6$) and $I = 0$.

The tower is obtained as follows:

$$
G_{q+2}(W) = W^1\overline{W}^1(W^1)^q \tag{6.12}
$$

where the massive graviton, described in eq. (6.12) has $E_0 = 5 + \frac{3}{2}q$ and $I = q/2$.

Note that the G_{q+2} polynomial, although not G-analytic, satisfies the constraint

$$
D_\alpha^1 D^{1\alpha} G_{q+2}(W) = 0 \tag{6.13}
$$

so that it corresponds to a short representation with quantized dimensions and highest weight given by $D(E_0 = 3 + 3I, 0, 0; I = q/2)$.

We call these multiplets, following [25], "intermediate short" because, although they have some missing states, they are not BPS in the sense of supersymmetry. In fact they do not form a ring under multiplication.

It is worthwhile to mention that the towers given by (6.7) and (6.12) correspond to the two isolated series of UIR's of the $F(4)$ superalgebra argued to exist in [12].

There are also long spin 2 multiplets containing 2^8 state where E_0 is not quantized and satisfies the bound $E_0 \geq 6$.

Finally let us make some comments on the role played by the flavour symmetry $\mathcal{G}$.

It is clear that, since the supersingleton W^1 is in a representation of $\mathcal{G}$ (other than the gauge group of the world-volume theory), the I_p and G_{q+2} polynomials will appear in the p-fold and $(q+2)$-fold tensor product representations of the $\mathcal{G}$ group. This representation is in general reducible, however the 1/2 BPS states must have a first component totally symmetric in the SU(2) indices and, therefore, only certains $\mathcal{G}$ representations survive.

Moreover in the $(W^1)^2$ multiplet, corresponding to the massless $\mathcal{G}$-gauge vector multiplets in AdS_6, we must pick up the adjoint representation $Adj\mathcal{G}$ and in $W^1\overline{W}^1$, corresponding to the graviton multiplet, we must pick up the $\mathcal{G}$ singlet representation.

However in principle there can be representations in the higher symmetric and antisymmetric products, and the conformal field theory should tell us which products remains, since the flavor symmetry depends on the specific dynamical model.

The states discussed in this paper are expected to appear [8, 9] in the K-K analysis of IIA massive supergravity on warped $AdS_6 \otimes S^4$. It is amusing that superconformal field theory largely predicts the spectrum just from symmetry cosiderations. What is new in the $F(4)$ theory is the fact that, since it is not a theory with maximal symmetry, it allows in principle some rich dynamics and more classes of short representations than the usual compactification on spheres.

The K-K reduction is related to the horizon geometry of the D4-branes in a D8-brane background in presence of D0-branes [5, 8].

Conformal theories at fixed points of $5d$ gauge theories exist [11] which exibit global symmetries $E_{N_f+1} \supset \mathrm{SO}(2N_f) \otimes \mathrm{U}(1)$, where $N_f \geq 1$ is the number of flavors (N_f D8-branes) and U(1) is the "instantons charge" (dual to the D0-brane charge).

The E exceptional series therefore unifies perturbative and non-perturbative series of the gauge theory.

It is natural to conjecture that a conformal fixed point $5d$ theory can be described by a singleton supermultiplet in the fundamental rep. of E_{N_f+1}. For the exceptional groups $N_f \geq 5$ these are the **27** of E_6, the **56** of E_7 and the **248** of E_8 which are respectively complex, pseudo-real and real. The E_7 case was considered in ref. [5]. States coming from wrapped D8-branes will carry a non-trivial representation of $\mathrm{SO}(2N_f)$, which, together with some other states, must complete representations of E_{N_f+1}. It is possible that from the knowledge of $\mathrm{SO}(2N_f)$ quantum numbers of supergravity in D4-D8 background one may infer the spectrum of E_{N_f+1} representations and then to realize these states in terms of boundary composite conformal operators.

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